Essential Mathematics for Global Leaders I

Spring 2019

Statistics

Lecture 8: 2019 June 17-24

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Where are we ? Today’s plan

PART II: Statistical inference (推計統計学)

4. Null Hypothesis Significant Test (NHST) 帰無仮説検定

4.1 Concepts and 1st example: z-test

4.2 chi2 and sample variance （カイ二乗と標本分散）
4.3 the Student t-test （Student t-検定)
4.4 Two-sample t-test (=variance) 対応のないt検定
4.5 Paired difference sampling 対応のあるデータ
4.6 Comparing two population variances: F-test

4.7 chi2-test (goodness-of-fit)カイ二乗(簡単な適合度検定)
4.8 chi2-test of independence 独立性のカイ二乗検定
4.9 One-way ANOVA (F-test) 一元配置分散分析 (F検定)

Chapter 4: NHST
Section 4.2 chi2 and sample variance
χ^2 distribution

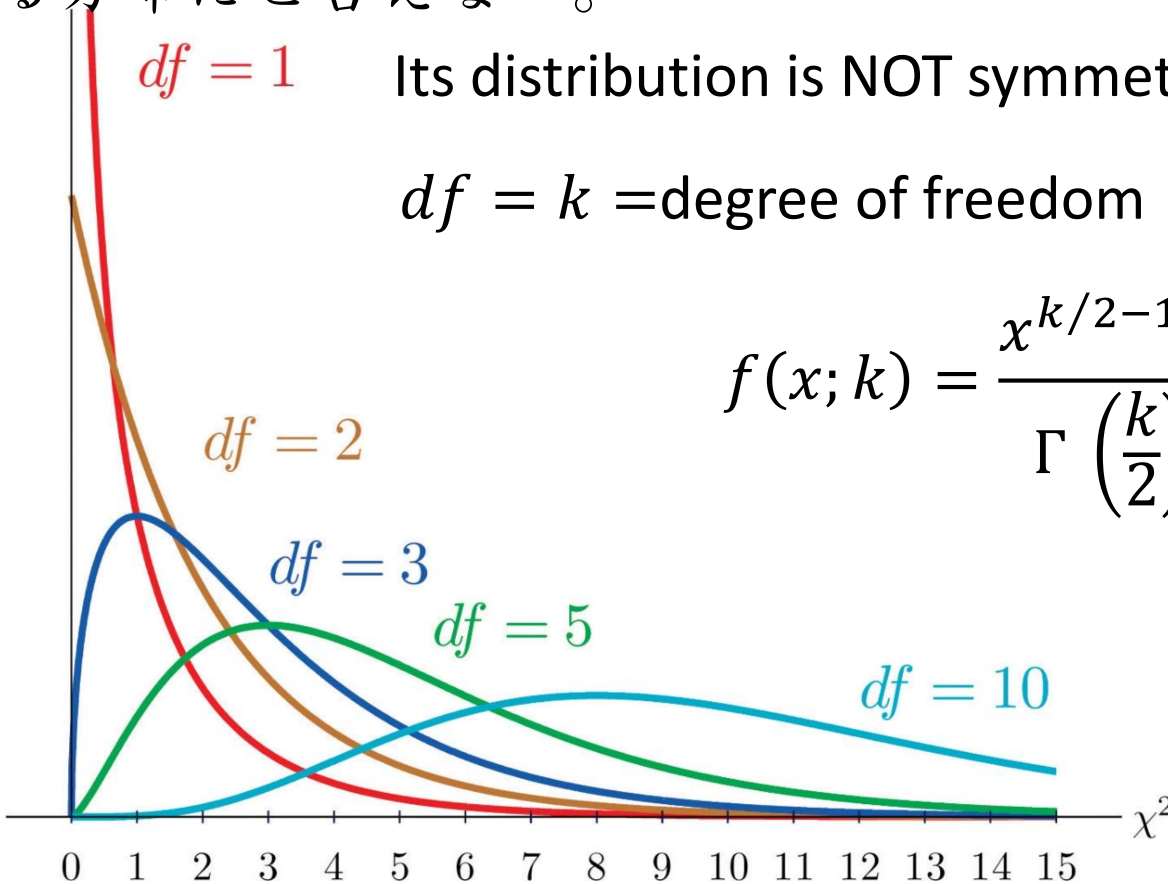
- $X_1, \ldots, X_k$ i.i.d.r.v. from the standard normal $N(0,1)$.

Theorem 1: The random variable $Q_k = X_1^2 + X_2^2 + \cdots + X_k^2$ follows a χ^2-distribution with k degrees of freedom (自由度)

- For any $k \geq 1$, $\int_0^\infty x^{k/2-1} e^{-x/2}\, dx = \Gamma(k/2) 2^{k/2}$.
- **Definition**: χ_k^2 has for pdf:
  - $f(x;k) = \frac{x^{k/2-1} e^{-x/2}}{\Gamma\left(\frac{k}{2}\right) 2^{k/2}}$ $if\ x \geq 0 \quad and\ 0\ if\ x < 0.$
  - $E(Q_k) = \int_0^\infty x\, f(x;k) dx = k$
  - $Var(Q_k) = E\left(Q_k^2\right) - E(Q_k)^2 = \int_0^\infty x^2\, f(x;k)\, dx - k^2$
    $= 2k$

Some pdf plots of the chi2-distrib

- Used a lot in statistics but does not describe natural phenomena like the normal, or exponential distributions. 統計学においてよく使われているが、自然な現象を記述する分布だと言えない。
- Its distribution is NOT symmetric



$df = k$ =degree of freedom 自由度

$$f(x;k) = \frac{x^{k/2-1}e^{-x/2}}{\Gamma\left(\frac{k}{2}\right)2^{k/2}}$$

Sample variance and χ^2 distribution

- Sample $X_1, \dots, X_n$ i.i.d.r.v. from a normally distributed population $N(\mu, \sigma^2)$.
- Unbiased 不偏 sample variance is ([Lecture 6 slide 10])

$$s_n^2 = \frac{n}{n-1}\,\bar{\sigma}_{\bar{X}}^2 = \frac{1}{n-1}\left((X_1 - \bar{X})^2 + \cdots (X_n - \bar{X})^2\right)$$

- $E(s_n^2) = \sigma^2$ *(over sampling distrib. 標本分布上に)*

Theorem 2 (Helmert/Cochran): The (unbiased) sample variance s_n^2 follows a χ_{n-1}^2 distribution.

$$(n-1)\frac{s_n^2}{\sigma^2} \sim \chi_{n-1}^2$$

Approximation (近似) of σ^2 by s_n^2: one sample is enough ?

➢ $Var_{samp.distr.}(s_n^2) = \frac{\sigma^4}{(n-1)^2} Var(\chi_{n-1}^2) = \frac{2\sigma^4}{n-1}$

- When n is large the variance of the sample variance s_n^2 is very small (=samples whose sample variance are far away from the population variance σ are rare). 標本大きさnが増加するにつれて、標本分散s_n^2の分散は小さくなる（=母分散σ^2から離れている標本分散のある標本が珍しい)
- Therefore, when n is large the sample variance is almost surely a good approximation of the variance of the population. 標本大きさnが大きいときに、標本分散は母集団の分散のよい近時を与えることはほとんど確実にある。

Chi2-test for variance (small sample)

- **Assumption**: sample $X_1, \dots, X_n$ from a $N(\mu, \sigma^2)$ normally distributed population.
- Don't care about μ, want to have some idea about σ.
- **Null hypothesis H_0**: $\sigma = \sigma_0$ (σ_0 fixed by you)
- **Test statistic**: $x^2 = \frac{(n-1)s^2}{\sigma_0^2}$, $f(x^2|H_0)$ is the pdf of χ^2_{n-1}
- **P-values**: (i) $p_r = P(X^2 > x^2)$ (right-sided: $H_A: \sigma > \sigma_0$)
  (ii) $p_l = P(X^2 < x^2)$ (left-sided: $H_A: \sigma < \sigma_0$)
  (iii) $2\min\{p_r, p_l\} = 2\min\{p_r, 1 - p_r\}$ (2-sided: $H_A: \sigma \neq \sigma_0$)
- **Reject H_0 if**: (i) $p_r \leq \alpha$
  (ii) $p_l \leq \alpha$
  (iii) $p_r \leq \alpha/2$ or $p_l \leq \alpha/2$ (same as $2\min\{p_r, 1 - p_r\} \leq \alpha$)

Practice example (lightbulbs = 白熱電球)

- Lightbulbs manufacturer wants to estimate the lifetime (in hours) of their product.
- Assumption: lifetime follows $N(\mu, \sigma^2)$.
- They measured the lifetime of 5 lightbulbs and got x_1 =983, x_2 =1063, x_3 =1241, x_4 =1040, x_5 =1103

1. Compute the sample mean $\bar{x}$ and (unbiased) sample variance s^2
2. Test the assumption $H_0 : \sigma^2 = 4000$ with a one-sided and 2-sided chi2-test.

Answer: $\bar{x} = 1086, s^2 = \frac{1}{4}(103^2 + 23^2 + 155^2 + 46^2 + 17^2) = 37568.$

$x^2 = \frac{4s^2}{4000} = 37.568$ $(P(X^2 > x^2) \approx 0.00$. Reject if 1-sided, reject if 2-sided at significance level 0.001.

Chapter 4: NHST
Section 4.3: the Student t-test
Back to the Z-test and large sample

- Review of z-test learned in Lecture 7, slide 11.
  - Need the population mean to be normally distributed
    母集団の対象パラメータは正規分布$N(\mu, \sigma^2)$に従う.
  - Need to know the population variance
    母分散σ^2が既存だと必要である。

➢ If the sample is large, no need of these assumptions:
もしも標本サイズが十分大きいと、上記の仮定がだいぶ不要になる。

➢ Large-sample usually means at least $n \geq 30{\sim}50$
  ➢ Why ? Because then the CLT applies 中心極限定理は有効になるからである

Z-test for the mean: Large-sample (unknown distribution and variance)

- CLT: $\bar{X} \approx N\left(\mu, \frac{\sigma^2}{n}\right)$ (no need to assume that the population is normally distributed) 対象となる母数の分布は正規分布に従うと仮定しなくてもいい).
- Moreover the sample variance s^2 is a good approximation of the population variance σ^2. さらに、標本分散s^2は母分散のよい近似を与える。
  - ➢ We do not need to know σ and can use instead s.
  - ➢ 母標準偏差σが未知でも構わない、その代わりに標本偏差s^2を使うことができる。

➢ Possible to use $z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ instead of $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$ to estimate the mean even for non-normally distributed population

Small sample: t-test
1) Student's t distribution

William Gosset

- $Z = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$
- $Q_n = (n-1)\frac{S^2}{\sigma^2} \sim \chi^2_{n-1}$ (see Theorem 2 slide 4)
- **Theorem 3** (Gosset = Student, Fisher)

$$T = \frac{Z}{\sqrt{Q_n/n-1}} = \frac{\bar{X}_n - \mu}{S/\sqrt{n}} \sim t_{n-1}$$

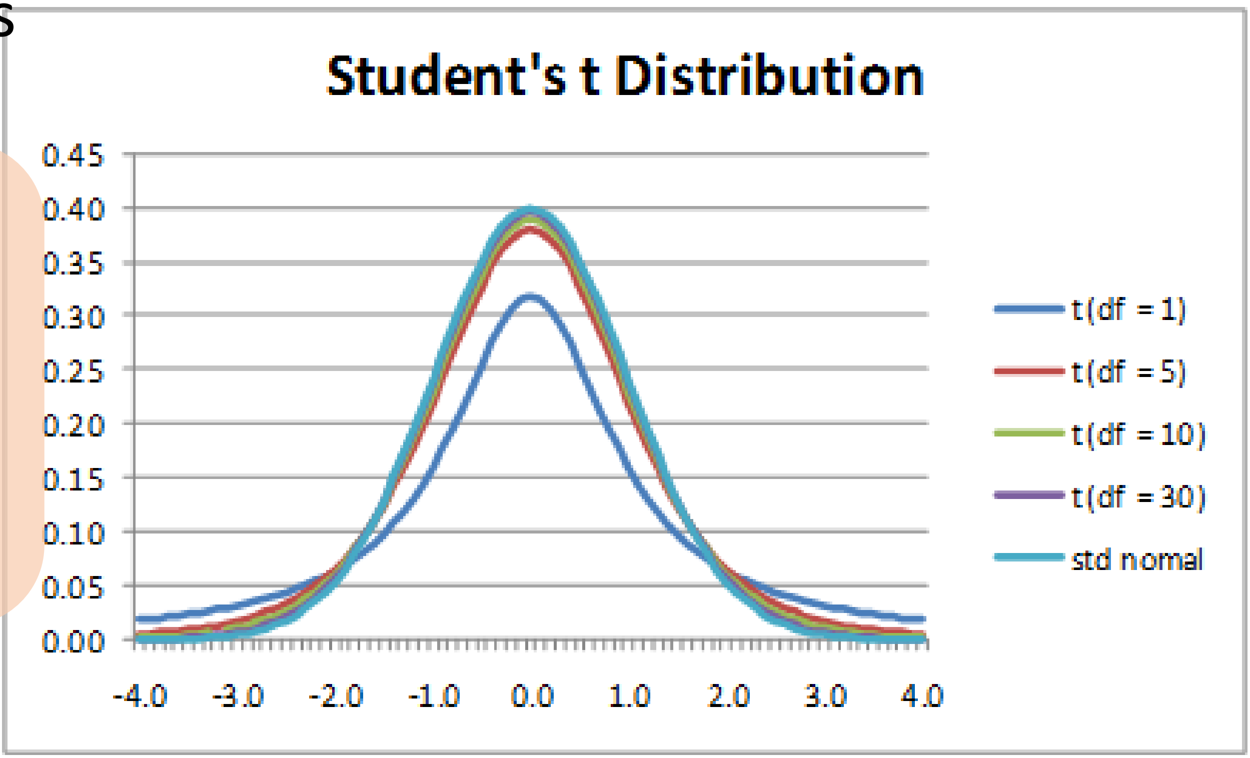
  - Where t_{n-1} is the Student's t-distribution.
  - $n-1$ is the degree of freedom (df) = 自由度
- **Remark**: the unknown variance σ^2 has been canceled out !
- The pdf is : $f(x; n-1) = \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\left(1 + \frac{x^2}{n-1}\right)^{-n/2}$

Don't care

Small sample: t-test. 2) T-distribution (II)

$$f(x; n-1) = \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\left(1 + \frac{x^2}{n-1}\right)^{-n/2}, \lim_{n\to\infty} f(x;n) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

- The distribution is symmetric.
- When the degree of freedom df increases, the distribution approaches the $N(0,1)$.
- $E(T_n) = 0$
- $Var(T_n) = \frac{n}{n-2}$



1-sample t-test for the mean (unknown variance, small sample)

- **Data**: we assume normal data with both μ and σ unknown: $x_1, x_2, \ldots, x_n \sim N(\mu, \sigma^2)$.
- **Null hypothesis H_0**: $\mu = \mu_0$ for some specific value μ_0.
- **Test statistic**:

$$t = \frac{\bar{x}_n - \mu_0}{s/\sqrt{n}} \text{ (Studentized mean)}$$

where $s^2 = \frac{1}{n-1}\left((x_1 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2\right)$ is the (unbiased) sample variance.

- **Null distribution**: $f(t \mid H_0)$ is the pdf of $T \sim t_{n-1}$, the t distribution with n − 1 degrees of freedom.
- **p-value**:
  two-sided $p = P(|T| > |t|)$
  Left-sided: $p = P(T < t)$
  Right-sided: $p = P(T > t)$

t-test works for small sample

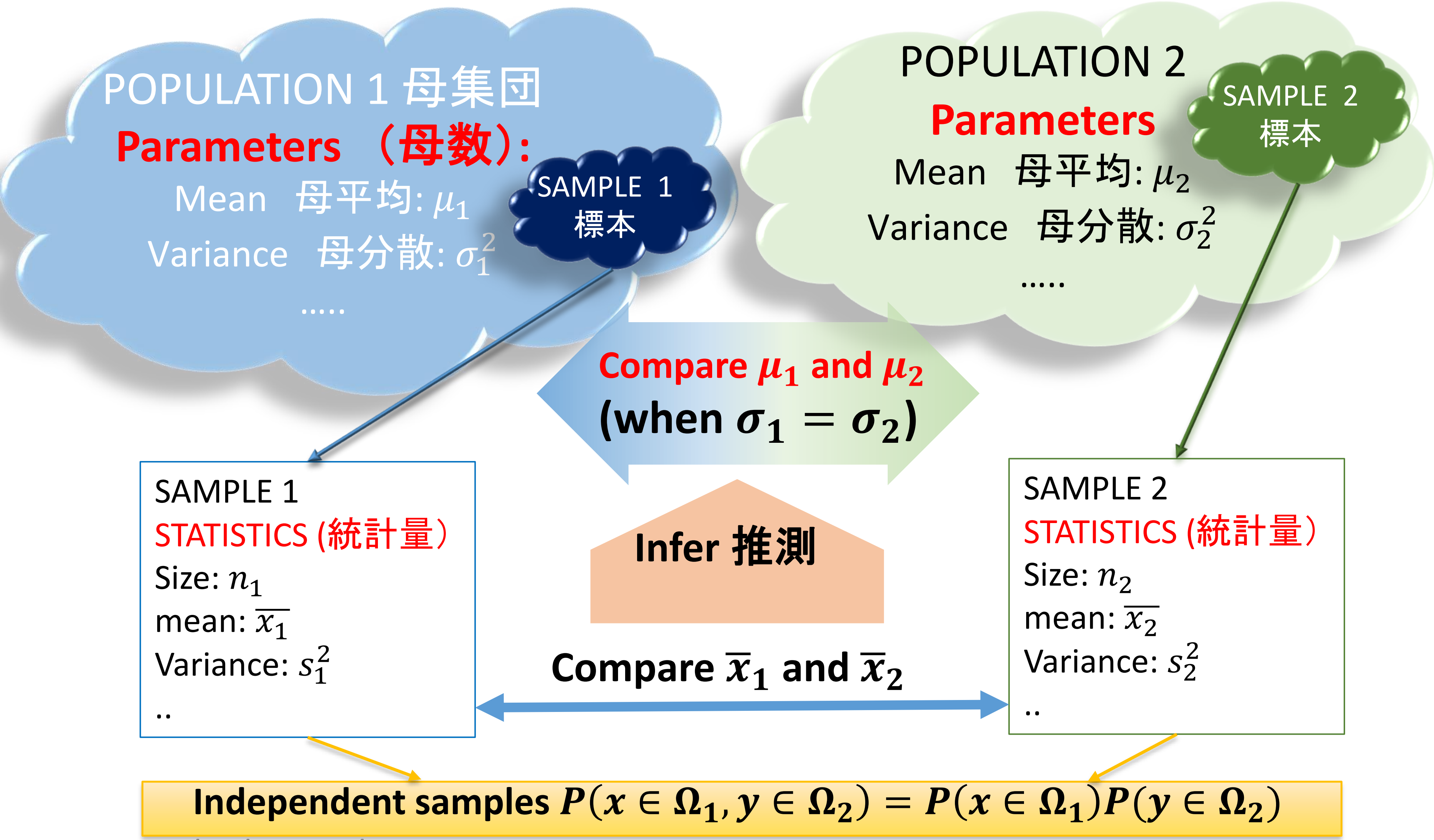
- When the sample is small:
  - The CLT does not hold and we cannot say that $\bar{X} \approx N\left(\mu, \frac{\sigma^2}{n}\right)$ 中心極限定理が適用されない。
  - ✓However, if the population is (approximately) normally distributed then the r.v. sample mean $\bar{X}$ is (approx.) normally distributed.　標本平均はほぼ正規分布で近似できる。
  - But, the sample variance S^2 may not be a good approximation of the population variance σ^2. しかし、(不偏の)標本分散S^2は母分散σ^2からよい近似できないかもしれない。
  - ✓Better use a t-test ☞ no population variance required in the t-statistic.　ここで母分散が使われていない.
- **Question:** If we don't know σ, why not using a chi2-test (page 8) to estimate it?
- *Answer*: test says whether $\sigma \neq \sigma_0$ for some estimated value σ_0. Not that $\sigma = \sigma_0$ (unless we can compute the power). Population must be very close to be normally distributed.

Practice: z and 1-sample t-test

- For both questions use significance level $\alpha = .05$.
- Assume the data 2, 4, 4, 10 is drawn from a $N(\mu, \sigma^2)$.
- $H_0: \mu = 0$ $\qquad$ $H_A:\ \mu \neq 0$

1. Assume $\sigma^2 = 16$ is known and test H_0 against H_A.
2. Now assume σ^2 is unknown and test H_0 against H_A

Chapter 4: NHST
Section 4.4 Two-sample t-test



Two-sample t-test ($\sigma_1 = \sigma_2$) in practice

- **Data**: we assume normal data with μ_1, μ_2 and **(same)** σ unknown:
  $x_1, \ldots, x_{n_1} \sim N(\mu_1, \sigma^2)$, $\quad y_1, \ldots, y_{n_2} \sim N(\mu_2, \sigma^2)$
- **Null hypothesis H_0**: $\mu_1 = \mu_2$.
- **Pooled variance**: $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$
  where s_1^2 (resp. s_2^2) is the sample variance of the x (resp y) sample.
- **Test statistic**: $$t = \frac{\bar{x} - \bar{y}}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$
- **Null distribution**: $f(t \mid H_0)$ is the pdf of $T_{n_1+n_2-2} \sim t_{n_1+n_2-2}$
- **P-value:** $P\left(T_{n_1+n_2-2} > t\right)$ (right-sided, H_A: $\mu_1 > \mu_2$)
  $P\left(T_{n_1+n_2-2} < t\right)$ (left-sided, H_A: $\mu_1 < \mu_2$)
  $P\left(\left|T_{n_1+n_2-2}\right| > |t|\right)$ (two-sided, H_A: $\mu_1 \neq \mu_2$)

Practice of the 2-sample t-test for the mean

Real data from 1408 women admitted to a maternity hospital for
実のデータによると、産科病院に入院した1408人の女の人を二つの理由で以下のように分かれている：

(i) medical (booked) reasons or through
医療上の（予約）理由で

または

(ii) unbooked emergency admission.
（予約されない）急患診療.

The duration of pregnancy is measured in complete weeks from the beginning of the last menstrual period.
最後の月経期から週数によって妊娠期間が計られる。

…続き

- (i) Medical: 775 obs. with $\overline{x}$ = 39.08 and s^2 =7.77.
- (ii) Emergency: 633 obs. with $\overline{x}$ = 39.60 and s^2 =4.95

1. Set up and run a two-sample t-test to investigate whether the duration differs for the two groups.
2. What assumptions did you make?

Some Remarks about 2-sample t-test

- Possible to use two-sample Z-test as well if:
  - Sample sizes n_1 and n_2 are large ≥30~50 and the populations variances σ_1^2 and σ_2^2 are known
    標本数n_1とn_2がともに３０～５０以上であり、母集団分散σ_1^2とσ_2^2が既存であるとき
  - Or both population follows a normal distribution and σ_1^2 and σ_2^2 are known 二つの母集団の分布が正規分布に従い、σ_1^2とσ_2^2が既存であるとき

$$Z = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

- **Welch's test**: case where $\sigma_1 \neq \sigma_2$ (未知、unknown)
  - Then $t = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{S_1^2/n_1 + S_2^2/n_2}}$ follows t-distribution
  - Degree of freedom ! $df = \left\lfloor \frac{(S_1^2/n_1 + S_2^2/n_2)^2}{(S_1^2/n_1)^2/(n_1-1) + (S_2^2/n_2)^2/(n_2-1)} \right\rfloor$

Control (placebo) group vs treatment group **Example "The bad scientist…**

- Suppose that a scientist Jerry is terrible at designing effective treatment but he always carefully randomly divides his patients into control and treatment groups.
  科学者Jerryは効き目のある治療を設計するのが下手だが、常に患者を独立無作為にコントロール（対照群）と治療群の2つに分ける。
- His null hypothesis H_0 is that the treatment is no better than the placebo.
  帰無仮説は治療はプラセボより効くことはない。
- He uses a significance level of α =0.05.
- If his p-value is less than α he publishes a paper claiming the treatment is significantly better than a placebo. p値が0.05の有意水準より小さいと、治療はプラセボよりも有意に効くと判断し、論文を投稿し出版する。

1. Since his treatments are never, in fact, effective what percentage of his experiments result in published papers?
   Jerryが行った実験が科学論文になった割合を計算するにはどうすればよいか。
   Hint: *Think about significance level and type of error.*
2. What percentage of his published papers describe treatments that are better than placebo?
   実際に効能のある治療を記述する科学論文の割合はどのくらいか.

Answer: 1. 5%. $\alpha = P(type\ I\ error) = P(reject\ a\ correct\ H_0) = 5\%$

2. None.

..and the good scientist”

Jenna is a genius at designing treatments, all her proposed treatments are effective. She always tests her new treatment with control/treatment group,
Jennaは治療の設計に対して才能があり、提案した治療はすべて効能がある。帰無仮説を彼女はコントロール（対照群）と治療群に対すて新しくテストし、

帰無仮説を

and set the null hypothesis to be

“H_0: the treatment is not more effective than placebo”
「治療はプラセボより効くことはない」

and runs a two-sample t-test at significance level $\alpha = 0.05$.
をとし、0.05有意水準　2つの標本t-検定を行う。

She publishes a paper if her p-value<α.
p値<0.05成り立ったら論文を投稿し出版する。

1. How could you determine what percentage of her experiments result in publications?
Jennaが行った実験が科学論文になった割合を計算するにはどうすればよいか。
<u>Hint:</u> *Think about significance level and type of error.*
2. What percentage of her published papers describe effective treatments?
実際に効能のある治療を記述する科学論文の割合はどのくらいか。

<u>Answer:</u> 1. 1 – p(type II error) = power of the treatment.
If a tiny better than placebo, then nearly 5%. If much better than placebo then virtually 100%.

2. All.

Chapter 4: NHST
Section 4.5 Paired difference sampling
対応のあるデータ

- **Example**: Test a teaching method on slow-learners kids.
  学業遅滞児に対する指導法を検定する。
- reading IQs: measure ability to learn to read for kids ($\mu = 100$). 子供に関して、読書能力を獲得できることをはかるもの： 読書IQ
- Let 8 pairs (n=16) of slow-learners kids.
  学業遅滞児の8ペア(16子)を考える。
- The two kids in each pair have similar reading IQs.
  各ペアにいる二人の子は同等な読書IQがある。
- For each pair すべて子のペアに対して
  - A kid is randomly selected to learn a new method (N)
    新指導法を学ぶ子を無作為に選ぶ。
  - The other kid learns the standard method (S)
    他の子は標準法を学ぶ。

Example: Test results

Pair Nb	New (N) Method	Standard Method (S)
1	77	72
2	74	68
3	82	76
4	73	68
5	87	84
6	69	68
7	66	61
8	80	76

- 2-sample t-test:
- $\bar{x}_N = 76, \quad s_N = 6.93$
- $\bar{x}_S = 71.63, \quad s_S = 7.01$
- Test: H_0: New method is not better than Standard. $\mu_N = \mu_S$
  H_A: New method is better than the Standard. $\mu_N > \mu_S$
- Pooled sample: $s_P^2 = \frac{(n_1-1)^2 s_N^2 + (n_2-1)^2 s_S^2}{n_1+n_2-2} = \frac{7\cdot 6.93^2 + 7\cdot 7.01^2}{8+8-2}$
  $s_P^2 = 48.55, s_P \approx 6.9687,$
- $$t = \frac{\bar{x}_N - \bar{x}_S}{s_P \cdot \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{4.37}{3.4823} \approx 1.26$$
- T-table (df=14): 1-sided p-value > 0.1
  ☛ don’t reject at 0.1 significance level

Example (II): something is wrong ?

- However, the data clearly suggests that the new method is better than the standard one.
  ただし、データによると新方法は標準法よりも効果があるとはっきり推定できる。
- What's wrong ?
  何がおかしい？

➢ The samples on the two groups are not independent!!
  二つのグループの標本が独立ではない

➢ The two-sample t-test is not valid, the pooled variance s_P^2 is large compared to the small sample means difference $\bar{x}_N - \bar{x}_S$. 合併分散s_P^2は標本平均の差 $\bar{x}_N - \bar{x}_S$に比して大きすぎ、妥当ではない。
  - ➢ The test cannot detect such small differences.
    検定はこのような小さな差を検出できない

- What can we do?

➢ Use a 1-sample t-test on $\mu_D = \mu_N - \mu_S$.

Example (III): paired differences 一対比較

Pair Nb	New Method	Standard Method	Difference (N-S)
1	77	72	5
2	74	68	6
3	82	76	6
4	73	68	5
5	87	84	3
6	69	68	1
7	66	61	5
8	80	76	4

- $H_0: \mu_D = 0 \ (\mu_N - \mu_S = 0\ ) \qquad H_A: \mu_D > 0 \ (\mu_N > \mu_S)$
- 1-sample t-test: $t = \frac{\bar{x}_D - 0}{s_D \sqrt{n_D}} = \frac{4.375}{1.685/\sqrt{8}} = 7.34$
- T-table (df=$n_D - 1$=7) gives: p-value <0.0005 **reject H_0**

Paired difference NHST 一対比較法

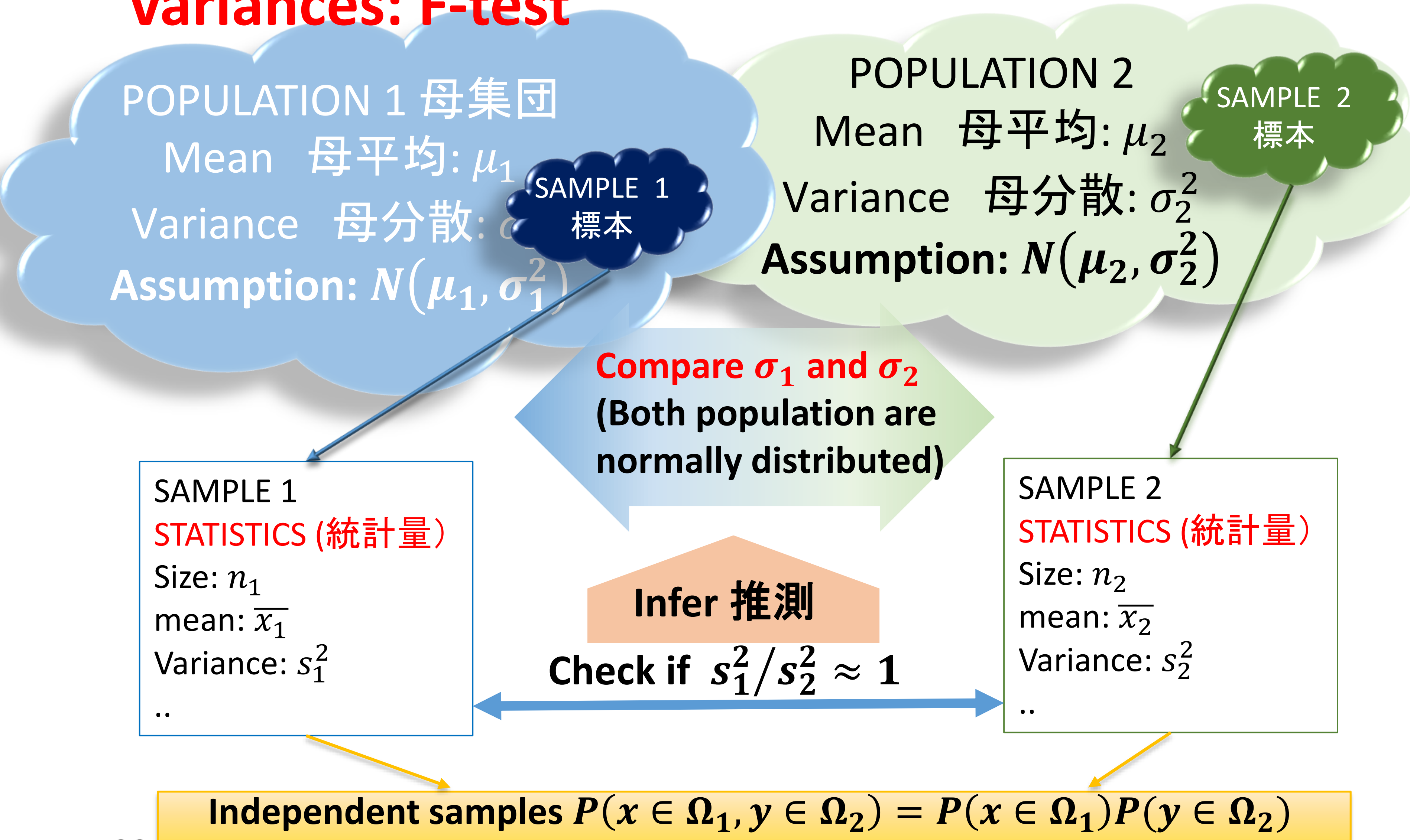
- H_0: $\mu_D = 0$ (or any constant D_0)
- H_A:$\mu_D \neq 0$ or < 0 or > 0 (or $\neq$ *or* $>$ *or* $< D_0$)
- **Large sample (z-test)**
  - Test statistic: $z = \frac{\bar{x}_D - D_0}{\sigma_D/\sqrt{n_D}} \approx \frac{\bar{x}_D - D_0}{S_D/\sqrt{n_D}}$ (follows $N(0,1)$ under H_0)
  - P-value: $P(Z > z)$ or $P(|Z| > |z|)$ or $P(Z < z)$

  Assumption
  - Sample size n_D is large (usually at least 30~50)
- **Small sample (t-test)**
  - Test statistic: $t = \frac{\bar{x}_D - D_0}{S_D/\sqrt{n_D}}$ (follows t_{n_D-1})
  - P-value: $P\left(T_{n_D-1} > t\right)$ or $P\left(\left|T_{n_D-1}\right| > |t|\right)$ or $P\left(T_{n_D-1} > t\right)$

  Assumption
  - population of difference is (approx.) $\sim N(\mu_D, \sigma_D^2)$

Chapter 4: NHST

Section 4.6 Comparing two population variances: F-test



The F-distribution

- **Theorem (Fisher-Snedecor)**
  If $X \sim \chi^2_{n_1}$ and $Y \sim \chi^2_{n_2}$ then the random variable $\frac{X/n_1}{Y/n_2}$ has a distribution called F with n_1 and n_2 degrees of freedom.
- The density function is complicated:

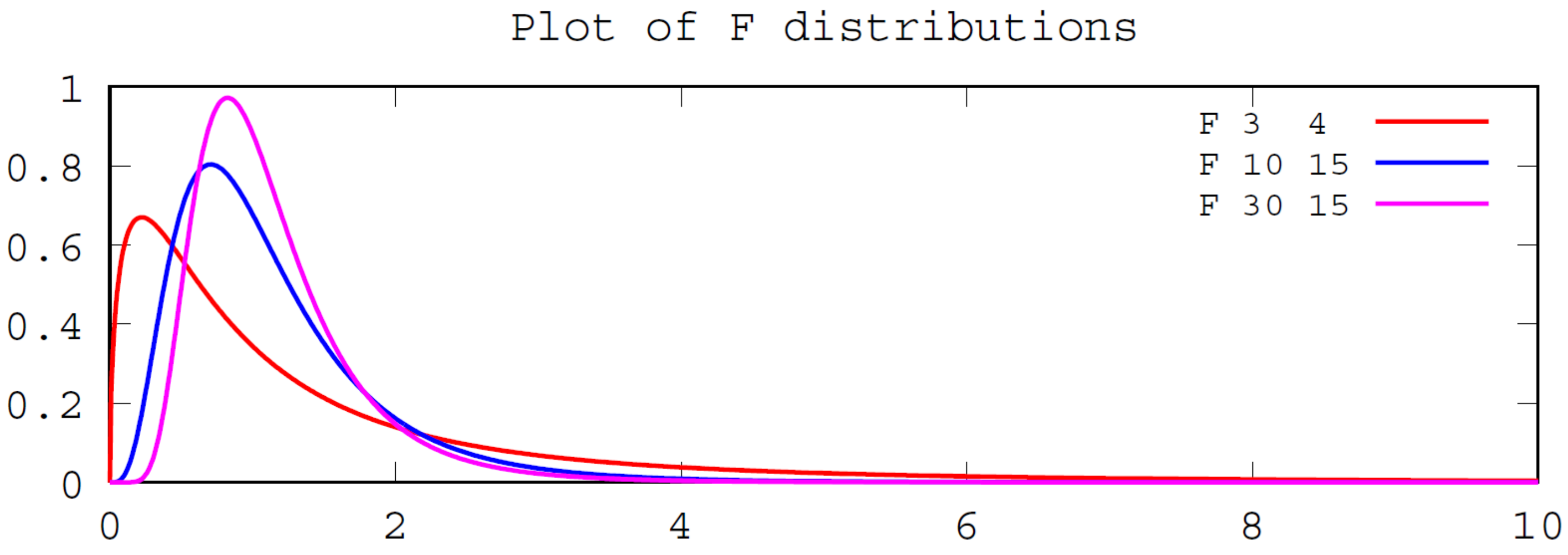
$$f(x; n_1, n_2) = \frac{(n_1/n_2)^{n_1/2} x^{n_1-1/2}}{B(n_1/2, n_2/2)} \left(1 + \frac{n_1}{n_2} x\right)^{-n_1-n_2/2}$$

  where $B(n_1/2, n_2/2) = \int_0^1 t^{n_1-2/2}(1-t)^{n_2-2/2}\ dt$

- If $F \sim F_{n_1,n_2}$ then $\mathrm{E}(F) = \frac{n_2}{n_2-2}$ and
- $Var(F) = \frac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)}$

F-distribution plot

- Not symmetric 対称ではない.
- Range [0; ∞) 領域



The F-test of equality of variance

- Recall the Theorem 1 (page 7) and Fisher-Snedecor's theorem (page 30)
  - $X = (n_1 - 1)\frac{s_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1}$ and $Y = (n_2 - 1)\frac{s_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$

  ➢ $\frac{X/n_1-1}{Y/n_2-1} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}$
- **Null hypothesis $\boldsymbol{H_0}$**: $\sigma_1 = \sigma_2$
- **Test statistic:** Under H_0 $\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \frac{s_1^2}{s_2^2}$ so $f = \frac{s_1^2}{s_2^2} \sim F_{n_1-1,n_2-1}$
- **p-value**: $P(F_{n_1-1,n_2-1} > f)$ (2-sided H_A: $\sigma_1 \neq \sigma_2$)

  (right-sided: H_A: $\sigma_1^2 > \sigma_2^2$, NOT ~~$\sigma_2^2 > \sigma_1^2$~~)
- Reject if $p < \alpha$ (1-sided) or $p < \alpha/2$ (2-sided).
- **Assumption:**
- Both populations are normally distributed, NOT only approximately.
- **So this test is not used a lot in practice** あまり使われていない

Example I

- Scores at a Mathematic MCQ test shows (source: 1998, *American Educational Research J.)*
- *数学の多項選択試験で得られた点数：*

	Males	Females
Sample size	1,764	1,739
Mean	48.4	48.9
Standard Deviation	12.96	11.85

- Test the hypothesis that the variance of the male's score is more variable than the females.
  男の点数の分散の方が女より変わりやすいという仮説を検定せよ。
- H_0: $\sigma_M = \sigma_F \quad f = s_M^2 / s_F^2 = 1.19612$
- 1-sided test using $F_{\infty,\infty}$-table: $\alpha = 0.01$ level gives $c_\alpha = 1 < f$ so we reject H_0 in favor of $\sigma_M > \sigma_F$
- (Using computer we find that $F_{1763,1738}(f) = .999909$)

Practice problem: back to slow learners

- Remember exercise page 26 on slow-learner kids.
- The t-test was made under the assumption that $\sigma_N = \sigma_S$.

1. Test this hypothesis at significance level $\alpha = 0.1$
2. Can we conclude that $\sigma_N = \sigma_S$?