Essential Mathematics for Global Leaders I

「みがかずば」の精神に基づきイノベーションを創出し続ける

理工系グローバルリーダーの育成

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博士課程教育リーディングプログラム

イノベーション創成科目

物理　数学　情報

理工学の基盤となる知識

- e-Physics
- e-Chemistry
- e-Mathematics
- e-Bioinformatic
- e-Computer Science
- e-Engineering and Technology

イノベーション創成に向けた幅広い知見

e- はessentialの略　　すべて英語で実施する

Statistics （統計学）

- Used in theoretical physics, chemistry (statistical mechanics), Computer Science (Machine Learning, Artificial Intelligence)
  理論物理学と化学（統計力学など)、コンピューターサイエンス（機械学習、人工知能）において利用される。
- Used in experimental sciences, clinical studies, social sciences etc.
  実験科学、臨床研究、社会学科などにおいても利用される。
- Also in Economy, Business, Mathematical Finance.
  経済学、ビジネス学、数理ファイナンス。

Statistics is Essential Mathematics

Basic principle of Statistical Inference

POPULATION 母集団

Parameters （母数）:

Mean 母平均: μ

Variance 母分散: σ^2

Stand. Dev : σ

Median 母中央値: M

.....

SAMPLE 標本

SAMPLE

STATISTICS (統計量)

mean: $\bar{x}$

Variance: s^2

Stand. Dev. s

Median: m

.....

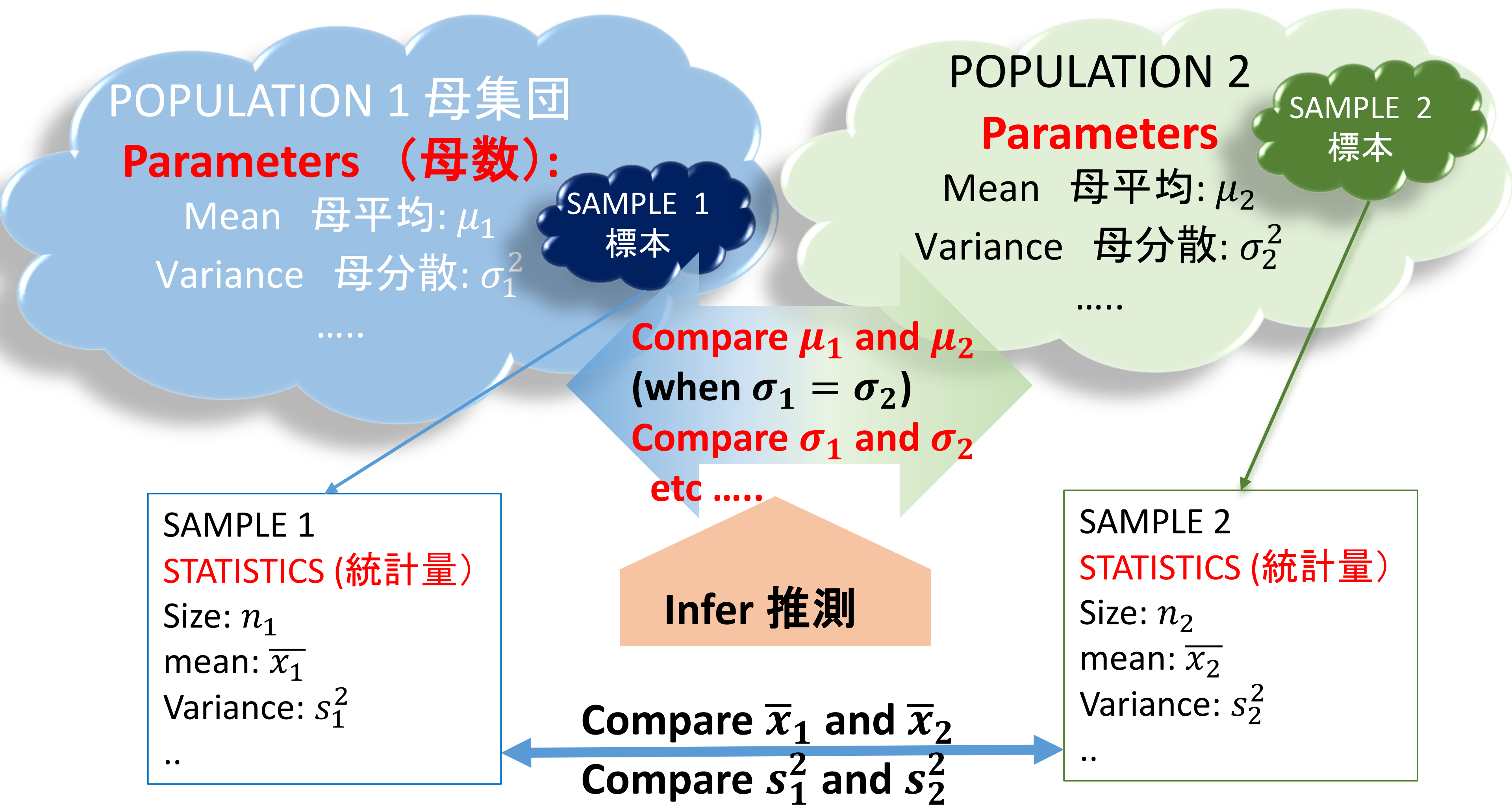
Infer 推測

- Population is too big to make direct measurements/observations.
一般的には母集団は大きすぎてそのパラメタを直接に測定するのは（時間的に、経済的になど)無理だ。
    - ☞Make the observations/measurements on a sample.
測定を標本上で行うこと。

Some questions raised/answered by Statistics

- What assumptions on the population can we reasonably make？　母集合に対する仮定の妥当性？
  Example: the parameter sought for is normally distributed
  求められるパラメタは正規分布に従う。
- Is it sensitive to this assumption?
  推定はこの仮定に対して感度が高いか低いか？
  Example: if the parameter sought for is not exactly normally distributed, is it a problem?　もし求められるパラメタは厳密に正規分布に従わないなら、問題が発生するか。
- How large shall the sample size be？
  標本はどれだけ大きければよいか？
- How the confidence/precision varies with the sample size？
  標本サイズに連れて、制度と信頼性をどのように変化するか

Inference about two populations



Spring 2019: Statistics - PLAN

PART I. Notions of Probability 必要な確率論

1. Basic Probability. 確率論基礎

2. Random Variables. 確率変数
   Important Examples (discrete and continuous).
   離散と連続確率変数の大事な例

3. Sampling Distribution 標本分布,
   Point estimation 点推定.
   Maximum likelihood 最尤推定.
   Central Limit Theorem 中心極限定理.

Spring 2019: Statistics - PLAN

PART II: Statistical inference (推計統計学)

4. Null Hypothesis Significant Test (NHST) 帰無仮説検定

- **One-sample tests**: z-test (正規分布)
  - chi2-test for variance (分散のカイ二乗検定)
  - small and large sample Student t-tests (一群のt検定)
- **Two (or more) -sample tests**:
  - t-test for comparing mean (equal variance or not 等分散・異分散の二群のt検定),
  - paired data (対応のあるデータ),
  - F-test or comparing variance (分散の比較F-test)
  - chi2-test (goodness-of-fit) カイ二乗 (適合度検定)
  - chi2 for independence (独立性のカイ二乗検定)
  - One-way ANOVA(F-test) 一元配置分散解析

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5. **Confidence Intervals. 信頼区間**
    - Comparison with NHST. 無帰検定と比較
    - Case of polling. 投票

6. **Linear regression. 線形回帰**

7. Joint distribution, Covariance, Correlation
    同時分布、共分散、相関関係

8. **Additional topics** (if time allows 残り時間次第):
    - Notion of non-parametric statistics
      ノンパラメトリック統計の概念
        - Example: Wilcoxon rank-sum & signed rank test.
        - 例：Wilcoxon順位和検定　と　符号付き順位検定
    - Multivariate analysis of variance (MANOVA)
      分散多変量解析

Why do we study probability???

- This is the language of Statistics 統計学の言語だ
- But it relies on Calculus.
  微分積分は確率論の根拠となる。

- Understanding probability ☛ Avoid common misunderstandings in the use of Statistics
  根本である確率論を理解することで統計学を使う際の誤解を回避できる。
- Statistics is a huge field, and in this class we only touch the tip of the iceberg.
  統計学は、大きな分野として本講義で氷山の一角だけを接する。
  But thanks to probability, it is easier to learn by oneself materials not taught in this class.
  確率論を用いて、教わらないものを自分自身で勉強するのはより簡単である。

About the class　授業の流れ

- Mathematical notions will be used the less possible
数学系の学生向けの授業だと想定しなく、数学の利用をできるだけ抑える。
- Focus on concepts and concrete examples（no mathematical proofs)
概念、具体例を中心にして授業を進める。
定理の証明をほとんどしない。
- Assessment 評価
  - Attendance. Short reports.
  出席。小レポート.
- During class: Lecture + problem solving.
講義+例題 or 練習問題を解く

Questions ?....let's start !

PART I. Notions of Probability 必要な確率論

1. Basic probability 確率論基礎 PLAN

1.1 Counting

- Venn diagrams, inclusion-exclusion principleベン図、包除原理
- Rule of product： $|S_1 \times S_2| = |S_1| \cdot |S_2|$
- Permutations of set (集合の置換)
- Permutations (順列) and Combinations (組合せ)
  Number of subsets in a set of cardinal n
  n個の集合からとれる部分集合の個数

1.2 Probability rules

- vocabulary: experiment, sample space, event, probability function (用語:確率論：実験、標本空間、事象、確率関数)

1.3 Conditional Probability, Independence, Bayes theorem

- 条件付き確率、独立性、ベイズの定理。

1. Basic Probability　確率論基礎

1.1 Counting　数える

- Motivating example: coin tossing(コイン投げ)（heads = H -> 表、　tails = T -> 裏）

- What is the probability of getting exactly 1 heads in 3 tosses of a fair coin? コインを３回投げて、表を1回ちょうど得るの確率は何か？

- **Answer:**
  - HTT, THT, TTH　3 possibilities
  - Over all possible 3 tosses: $2^3 = 8$.

$$P = \frac{|favourable\ outcomes|}{|all\ possibilities|} = \frac{3}{8}.$$

More difficult example.
Poker hands☞needs more tools !

- Deck of 52 cards (５２枚のトランプカード１組)
  - 13 *ranks* (ランク)： 2, 3, 4, ... , 9, 10, J, Q, K, A
  - 4 *suits* (スート) ： ♡, ♠, ♢, ♣

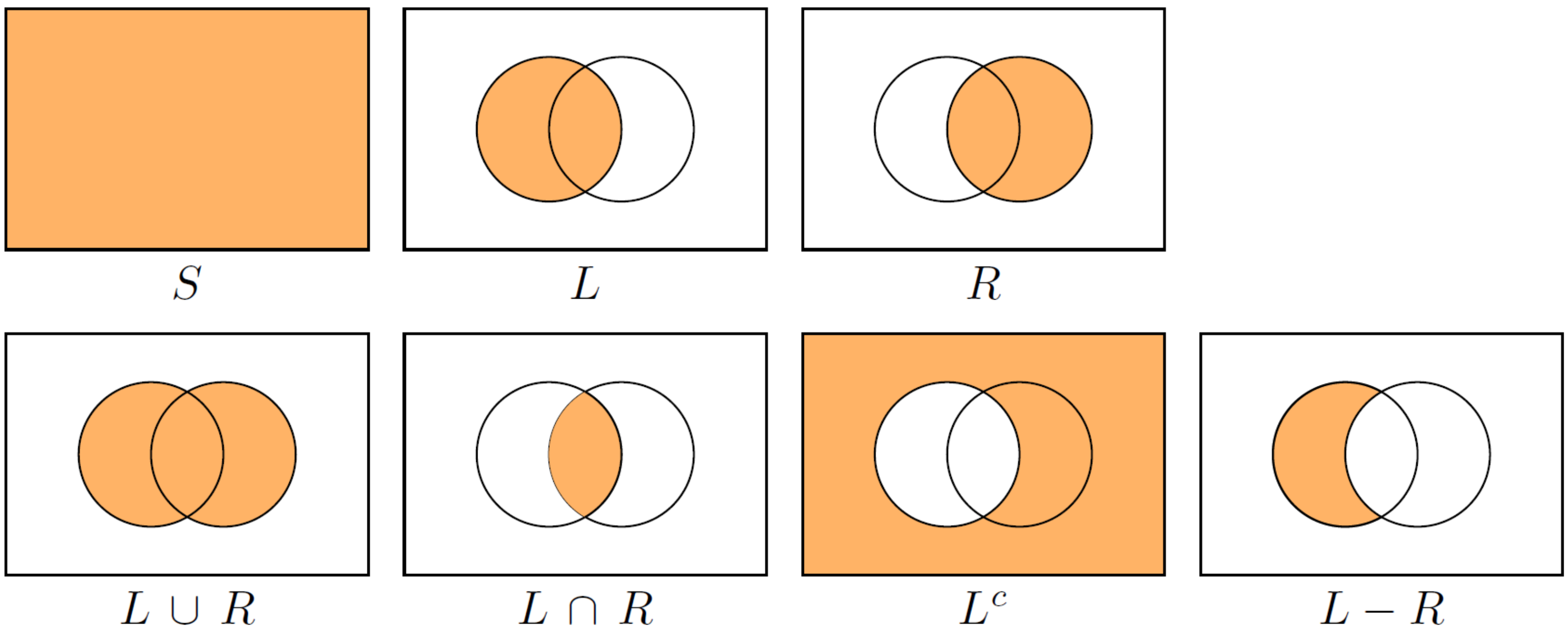
- Poker hands (5 cards)
  - A *one-pair* hand consists of (以下の条件を満たすハンドとはワンペアという）。
    ☛Two cards having the same rank.
    ☛Three other cards have different ranks.
  - Example: {2♡, 2♠, 5♡, 8♣, K♢}

- The probability p of a one-pair is

A) p < 5%　　B) 5% < p < 10%　　C) 20% < p < 40%　　D) p>40%

Inclusion-Exclusion Principle 包除原理

- Venn diagrams to represent sets: (集合を表現する)



- Inclusion–exclusion principle:
  $|L \cup R| = |L| + |R| - |L \cap R|$.

Example: inclusion-exclusion principle

- A music band (B) consists of singers (S) and guitar players (G)
  歌手(S)とギタリスト(G)からなるバンド(B)。
  - 7 people sing
  - 4 play guitar
  - 2 do both
- How many people in the band?
- $|B| = |S \cup G| = |S| + |G| - |S \cap G| = 7 + 4 - 2 = 9$

Rule of product

- Given two sets S and T, the set of pairs (s, t) of elements with $s \in S$ and $t \in T$ is denoted $S \times T$, and called product set. 集合Sと集合Tのペア要素(s, t)からなる集合を$S \times T$と書く、直積集合と呼ぶ。
  Example: 3 shirts , 4 pants = 12 outfits (一揃いの衣服)

- The number of pairs (s, t) in the product set $S \times T$ is equal to $|S| \cdot |T|$.

- Generalization (一般化) Number of elements $(s_1, s_2, \ldots, s_n) \in S_1 \times S_2 \times \cdots \times S_n$ is
$$|S_1| \cdot |S_2| \cdot \cdots \cdot |S_n|$$
- Example: number of secret codes of 4 digits between 0–9 ? 4 桁の暗証番号の個数？
- $S_1 = S_2 = S_3 = S_4 = \{0,1,2 \ldots, 9\}$ ☞ $10^4 = 10000.$

Another example.

- Example:
  DNA is made of sequences of nucleotides: A, C, G, T
- How many DNA sequences of length 3 are there?
  (i) 12 (ii) 24 (iii) 64 (iv) 81
- **Answer**: (iii) $4 \times 4 \times 4 = 64$

- How many DNA sequences of length 3 are there with no repeats?
  (i) 12 (ii) 24 (iii) 64 (iv) 81
- **Answer**: (ii) $4 \times 3 \times 2 = 24$

More difficult example

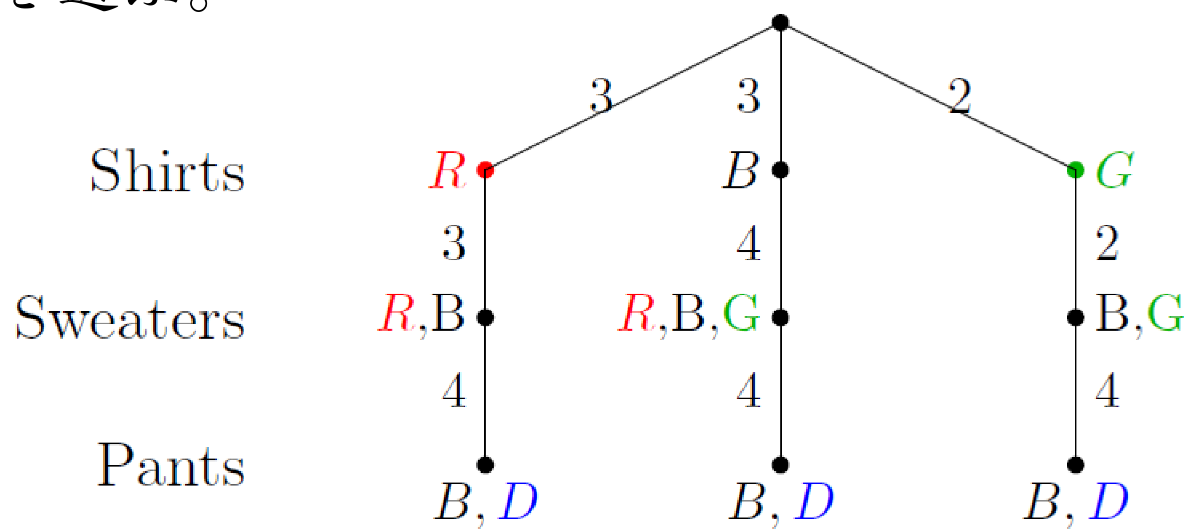
- I don' t wear green and red together
- Black or Denim goes with anything
- Here is my wardrobe (もち衣装)

Shirts: 3B, 3R, 2G; Sweaters 1B, 2R, 1G; Pants 2D,2B

- How many outfits can I wear?

Solution（解答）

- **Answer**: Suppose we choose shirts first, next the sweater and finally the pant.
まずシャーツを選んで、次にセーターを選んで、最後にパンツを選ぶ。



- Multiplying down the paths of the tree:
- Number of outfits = $(3 \times 3 \times 4) + (3 \times 4 \times 4) + (2 \times 2 \times 4) = 100$.

Permutations of a set 置換

- ‘abc’ and ‘cab’ are different permutations of $\{a, b, c\}$.
- How many ways can you do it?

- *abc, acb, bac, cab, bca, cba.* 6 different permutations. $6 = 3! = 3 \times 2 \times 1$

- In general, in $\{1,2,\ldots,n\}$ there are $n! = n \times n - 1 \times \cdots \times 2 \times 1$ permutations.

Permutations (順列) of k from a set of n

- Get all permutations of 3 things out of $\{a, b, c, d\}$ (ways of picking 3 elements in a set of size 4- **order matters**). 位数4の元からなる3–順列の個数？

$$
\begin{array}{cccccc}
abc & abd & acb & acd & adb & adc \\
bac & bad & bca & bcd & bda & bdc \\
cab & cad & cba & cbd & cda & cdb \\
dab & dac & dba & dbc & dca & dcb
\end{array}
$$

- We get $6 \times 4 = 24 = \frac{4!}{(4-3)!}$

- What about 7 from a set of 10? $\dfrac{10!}{(10-7)!} = \dfrac{10!}{3!}$

Combinations （組合せ）

- Choosing subsets out of a set of cardinal n (order doesn’t matter).
  位数n集合の部分集合の選び方(順番は関係ない).

Let S be a subset of $\{1, \dots, n-1\}$.
Then S is a subset of $\{1, \dots, n\}$, and $S \cup \{n\}$ as well.

☛ $|subsets\ of\ \{1, \dots, n\}| = 2|subsets\ of\ \{1, \dots, n-1\}|$

- And there are 2^n subsets of $\{1, \dots, n\}$.

Combinations of k from a set of n

- Give all combinations of 3 things out of $\{a, b, c, d\}$.
- Answer: $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$.

- Permutations (順列) and combinations (組合せ)

abc	acb	bac	bca	cab	cba	$\{a, b, c\}$
abd	adb	bad	bda	dab	dba	$\{a, b, d\}$
acd	adc	cad	cda	dac	dca	$\{a, c, d\}$
bcd	bdc	cbd	cdb	dbc	dcb	$\{b, c, d\}$

Permutations: P_3^4 Combinations: $C_3^4 = \binom{4}{3}$

$$C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{P_k^n}{k!}$$

Question (application)

a. Count the number of ways to get exactly 3 heads in 10 flips of a coin.
コインを１０回投げて、表が３回ちょうど出るのはいくつあるか？

b. For a fair coin, what is the probability of exactly 3 heads in 10 flips.
コインを１０回投げて、表が３回ちょうど出る確率はなにか？

Answer: a) Choose 3 out of 10: $\binom{10}{3}$

b) There are 2^{10} outcomes from 10 flips (this is the rule of products). Thus the probability is:

$$\binom{10}{3}/2^{10} = 120/1024 = 0.117$$

Back to poker hands

- Deck of 52 cards (５２枚のトランプカード１組)
  - 13 *ranks*: 2, 3, 4, ... , 9, 10, J, Q, K, A
  - 4 *suits*: ♡, ♠, ♢, ♣

- Poker hands (5 cards)
  - A *one-pair* hand consists of (以下の条件が成り立つとハンドはワンペアをなす)。
    ☛Two cards having the same rank.
    ☛Three other cards have different ranks.
  - Example: {2♡, 2♠, 5♡, 8♣, K♢}

- **Question**

a) How many 5 card hands have exactly a one-pair?
ワンペアがまさに一つを持つハンドはいくつか？

b) What is the probability of getting a one-pair poker hand?
ワンペアが持つハンドを出る確率はどれか？

Answer

- Can be solved using permutations (順列) or combinations (組合せ).

1. Be consistent (stick to combination if you use them)
   ~~節操~~ 整合的に説明すること
2. Break the problem into sequence of actions and use the rule of product.

Note that there are many ways to organize this.

その仕組みを行うように幾つかのやり方があり、ここで問題を分けて解く。

- **Combinations approach**
- **Permutations approach**

Combination approach

a) Count the number of one-pair hands. **Order doesn't matter.** $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 8\clubsuit, K\diamondsuit\}$

Action 1: Choose the rank of the pair: choosing one among 13 different ranks gives $\binom{13}{1}$ ways to do this

Action 2: *Choose* 2 cards from this rank: 4 cards/rank, choosing 2 among 4 -> $\binom{4}{2}$.

Action 3: *Choose* the 3 different ranks of the 3 other cards: 12 remaining ranks, so $\binom{12}{3}$ ways to do this.

Action 4: *Choose* 1 card in each of the 3 ranks: there are $\binom{4}{1}^3$ ways to do this.

Answer: (using the rule of product)

$$\binom{13}{1} \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3 = 1{,}098{,}240$$

Combination approach II

b) To compute the probability we have to stay consistent and count the combinations.
確率を評価する際に、節操して組合せを使って数える。

To make a 5 card hands we *choose* 5 cards out of 52, so there are

$$\binom{52}{5} = 2{,}598{,}960$$

possible hands. Therefore the probability is:

$$1{,}098{,}240\ /2{,}598{,}960\ = 0.42257$$

Next we use permutations.

Using permutations (訓順を用いて)

This is a little trickier. よい巧妙である。

a) Count the number of one-pairs, where order matters.

Action 1: *Choose* the position of the pair of same rank among the 5 positions possible: there are $\binom{5}{2}$ ways to do this. 同じランクのペアの位置を選ぶ。

Action 2: Put a card in the 1st position of the pair: 52 ways to do this トランプ一枚を最初の位置に置く。

Action 3: Put a card in the 2nd position; it remains 3 cards of the same rank, so 3 ways to do this.

Action 4 - 5 - 6 : Put a card of different rank: there are 48, then 44 and 40 ways to do this.

Using permutations II

- **Answer:** Using the rule of product
$$\binom{5}{2} \cdot 52 \cdot 3 \cdot 48 \cdot 44 \cdot 40 = 131{,}788{,}800$$

ways to deal a one-pair hand where we keep track of order.
ハンドの順番に注意するワンペアを数える。

b) There are
$$P_5^{52} = 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 = 311{,}875{,}200.$$

five card hands (where **order matters**).

Thus the probability of a one-pair hand is
$$131{,}788{,}800/311{,}875{,}200 = 0.42257$$

Homework　練習問題

1. Suggest sample spaces for the following experiments:次の実験に対して標本空間を提案せよ
    1. 3 dices are rolled and their sum is computed
       サイコロを三つ投げ、結果の和をとる。
    2. A Japanese person is chosen at random and is classified by age and gender.
       日本人をランダムで選び、年齢と性別で分類される。
    3. Two different integers are chosen at random between 1 and 10 and are listed in increasing order.
       1から１０まで異なる二つの整数をランダムで選び、増加順序で並べる。

2. How many passwords with 5 letters, 2 digits in any order are there in the following case:
    a. Repetitions are allowed
    b. Repetitions are not allowed

Homework. Problem 3

- After one pair the most common hands are two-pair and a three-of-a-kind: ワンペアの次に、最もありふれたハンドとはツーペアとスリー・オブ・ア・カインドである。
- *Two-pair:* 2 cards have one rank, two cards have another rank, and the last card has a third rank $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 5\clubsuit, K\diamondsuit\}$ 同じランクのある2枚と、このランクと異なるランクを共通する２枚と、最後の１枚のランクはその前の二つのランクと違う。
- *Three-of-a-kind:* 3 cards have one rank, and the remaining two cards have two different other ranks. $\{2\heartsuit, 2\spadesuit, 2\clubsuit, 5\clubsuit, K\diamondsuit\}$ 同じランクのあるカード３枚と、そのランクと異なるランク二つのある2枚。

☞**Calculate the probability of each type of hand? Which is most likely?** (ハンドがそれぞれの起こる確率を求めよ。どちらが最もありそうなハンドであるか？

1. Basic Probability 確率論

1.2 Probability rules 確率

Probability cast (配役)

- Experiment: a repeatable undetermined procedure.
  実験（試行）：繰り返せる、決定されていない手順。
- Sample space: Set of all possible outcomes Ω.
  標本空間：全ての可能な結果の集合。
- Event: a subset of the sample space.
  事象：標本の部分集合。
- Probability function, $P(\omega)$: gives the probability for each outcome $\omega \in \Omega$.
  各の結果に確率を割り当る。
  1. Probability is between 0 and 1.
  2. Total probability of all possible outcomes is 1.

Example:

- Experiment: toss a fair coin, report heads or tails.
  実験：コイン投げ。表か裏かが出たのを報告する。

- Sample space (標本空間)：$\Omega = \{H, T\}$.

- Probability function: $P(H) = 0.5, \ \ P(T) = 0.5$.

- Another experiment: toss a coin 2 times.
- Sample space: $\Omega = \{HH, HT, TH, TT\}$
- Probability function (using table):

Outcomes	***HH***	***HT***	***TH***	***HH***
Probabilty	1/4	1/4	1/4	1/4

理解を確かめる Events, sets and words

- Experiment: toss a coin 3 times.

- Which of the following is the event "exactly two heads"?

a. $A = \{THH, HTH, HHT, HHH\}$

b. $B = \{THH, HTH, HHT\}$

c. $C = \{HTH, THH\}$

(1) A (2) B (3) C (4)A or B

Answer: 2)B

The event "exactly two heads" is a *unique subset.*

理解を確かめる : Events, sets and words

- Experiment: toss a coin 3 times.
- Which of the following describes the event $\{THH, HTH, HHT\}$

1. "exactly one head" 表がちょうど１回
2. "exactly one tail" 裏がちょうど１回
3. "at most one tail" 裏が最大１回
4. None of the above どちらでも正しくない

Answer: 2) "exactly one tail"

(it could be "exactly two heads" as well)

理解を確かめる Events, sets and words

- Experiment: toss a coin 3 times.

- The events "exactly 2 heads" and "exactly 2 tails" are disjoint.
(。。。事象互いに素 =。。。交わりを持たない)
(1) True (2) False

- **Answer:**
True: $\{THH, HTH, HHT\} \cap \{TTH, THT, HTT\} = \emptyset$

- The event "at least 2 heads" implies the event "exactly two heads".
(1) True (2) False

- **Answer:** False. It is the other way around
$\{THH, HTH, HHT\} \subset \{THH, HTH, HHT, HHH\}$

Probability rules in maths notations 数学的な記号での確率法

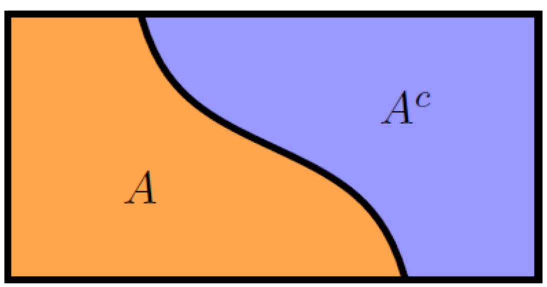
- Sample Space （標本空間）：$\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$.
- Outcome (結果）：$\omega \in \Omega$.
- Probability between 0 and 1: $0 \le P(\omega) \le 1$

- Total probability is 1:
  $\sum_{j=1}^{n} P(\omega_j) = 1, \qquad \sum_{\omega \in \Omega} P(\omega) = 1.$

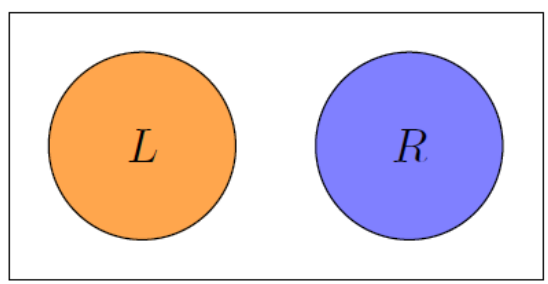
- Event (事象） $A \subset \Omega$: $P(A) = \sum_{\omega \in A} P(\omega)$.

Probability and set operations on events
確率と事象への作用

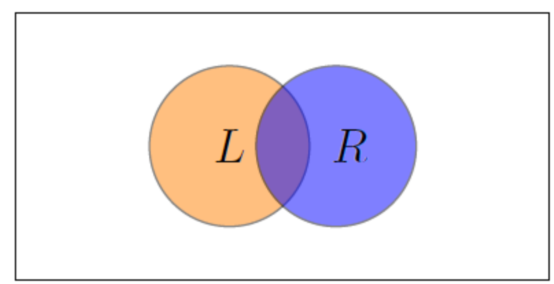
- Three events $A, L, R \subset \Omega$ （Ω: sample space）
- Rule 1: Complements （補集合） $P(A^c) = 1 - P(A)$.
- Rule 2: Disjoint events. （互いに素である事象）
  - If L and R are disjoint then
$$P(L \cup R) = P(L) + P(R).$$
- Rule 3: Inclusion–Exclusion principle （包除原理）
  - For any L and R:
$$P(L \cup R) = P(L) + P(R) - P(L \cap R).$$



Rule 1



Rule 2



Rule 3

Rule 1: example of application

- You roll a 20-sided die 9 times.
  正２０面体のサイコロを9回投げる。
- Event A: there is a match among the 9 outcomes.
  出た九つの結果の中で二つ以上は一致するという事象をAとする。
- Questions:

1. For this experiment how would you define the sample space, probability function, and event?
   この実験では、標本空間、確率関数と事象をどのように定義するか？
2. Evaluate the exact probability $P(A)$ that the event A occurs.
   事象Aが起こる確率$P(A)$を評価せよ。

Answer

- Sample space Ω: Sequences of 9 numbers between 1 and 20. １から２０間の長さ９の数列
- Cardinal of Ω: （Ωの個数） Use the rule of product！

$$|\Omega| = 20^9$$

- It is not very easy to count directly the nbr. of occurrences of the event A ⋯because there might be 2, 3, 4 ⋯ or 9 rolls that are equal.
直接事象Aの確率を評価するのは難しい。サイコロの結果が２つ、３つ、⋯、９つまで等しいである可能性があるから。

- But it is much easier for the complement A^c.
補集合の事象 A^cを計算するほうが簡単である。
- A^c: there is no match （等しい結果が無い）
- $|A^c| = 20 \cdot 19 \cdot \cdots \cdot 12 = P_{11}^{20} = 20!/11!$.
- $P(A) = 1 - P(A^c) = 1 - \frac{|A^c|}{|\Omega|} \approx 0.881$.

Rule 2: Example of application

- A group of 50 mice　（ネズミの５０匹）
- 20 have white hairs (W), and 25 have black eyes (B).
  毛皮が白い２０匹 (W)、目が黒い２５匹 (B)
- For a randomly chosen mouse, what is the range of possible values $p = P(W \cup B)$？
  ランダムで選択されたネズミに対して、確率$p = P(W \cup B)$の可能な範囲はなにか？

  a. $p \leq .4$
  b. $.4 \leq p \leq .5$
  c. ~~$.4 \leq p \leq .9$~~
  d. $.5 \leq p \leq .9$
  e. ~~$.5 \leq p$~~

Answer

- **Common sense (常識を使う):** there are at least 25 mice in $W \cup B$ and at most 45. Therefore the probability is:
$$0.5 = \frac{25}{50} \leq p \leq \frac{45}{50} = 0.9$$

- **Or use the inclusion-exclusion principle:**
また原理によると：
$$P(W \cup B) = P(W) + P(B) - P(W \cap B).$$

- Extreme cases are: $W \cap B = \emptyset$ and $W \subset B$.
- First case $P(W \cap B) = 0$ and 2nd case $P(W \cap B) = P(W) = \frac{20}{50}$.
- Giving: $0.9 - \frac{20}{50} = 0.5 \leq P(W \cup B) \leq 0.9$.

More difficult example with uncertainty
不確定性を持つより複雑な例

- Lucky Lucy has a coin that you' re quite sure is not fair.
  ラッキー・ルーシーはコインの一つを持って、あなた自身には公平だと信じない。
  - She will flip the coin twice （コインを２回投げる）
  - It' s your job to bet whether the outcomes will be the same *(HH, TT)* or different *(HT, TH)*.
    結果の両方が同じ*(HH,TT)*か異なる*(HT,TH)*かに賭ける

- Which should you choose?
  a. Same
  b. Different
  c. It doesn' t matter, same and different are equally likely

- Hint: Use the probability of heads p (and a little of algebra)
  表の確率をpとし、基礎的な代数を使って解ける。

Solution (解決)

Answer: a) same (same is more likely than different)

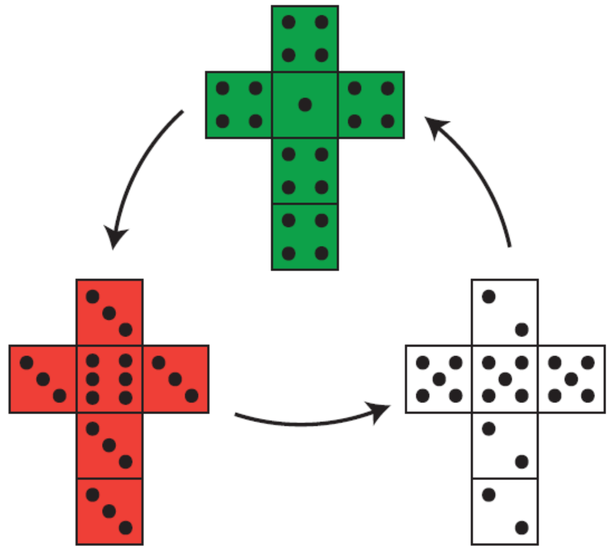
- $P(H) = p$ thus $P(T) = (1 - p) = q$.
- Since the flips are independent (Cf. Section 1.3) the probabilities multiply. This gives the following 2x2 table.
  コイントスは独立であるから、確率がかかる。以下の２行２列表を与える。

		H	T (2nd flip)
	1st flip H	p^2	pq
	T	pq	q^2

- $P(same) = p^2 + q^2, \quad P(diff) = 2pq.$
- Algebra: $(p - q)^2 = p^2 + q^2 - 2pq = P(same) - P(diff) > 0$
- So $P(same) > P(diff)$ if $p \neq q$.

Homework: Mari's dice

- Mari has three six-sided dice with unusual numbering.
  マリは異常な数付与のある６面サイコロを持つ。



- A game consists of two players each choosing a dice.
  二人のプレヤーのゲームではサイコロを選んで投げる。
  They roll once and the highest number wins.
  最大の結果は勝つ。
- **Question**: Which dice would you choose?

Hints:

A. You can proceed as follows for white and red.
   1. For red and white dice make the probability table.
      ワイトとレッドサイコロの確率表を作成する。
   2. Make a prob. table for the product sample space of red and white.
      レッドとワイトからなる直積標本空間の確率表を作成する。
   3. What is the probability that red beats white?
      レッドがワイトを負かす確率はなにか？

B. Then do the same for white and green dice.
C. And for the green and red dice as well.
D. Conclude which dice is the best for the game.

Answer to question A

- Red beats white is the event $(R > W)$:
- The answer is $P(R > W) = 7/12$. Find it by fulling the tables below.
- Probability table for white and red.

	Red die	
Outcomes	3	6
Probability		

	White die	
Outcomes	2	5
Probability		

- Both R and W outcomes in a 2x2 probability table:

		2	5	White
Red	3		5/12	
	6			

- Red entries: outcomes where red beat white.
- $P(R > W) = \qquad = \frac{7}{12}$.