MMA 数学特論I。多項式系のアルゴリズム: グレブナー基底 & 消去法
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Practice test IV: Resultant

- You can use any theorem, proposition or corollary of the class lectures, just by citing its number inside the corresponding lecture: (example: "Lect II, Cor. 1" refers to the Corollary 1 of Lecture II, that is the Primitive Element Theorem).

Exercise 1 Write the Sylvester matrix of the polynomials A and B:

A	$2x^2+x-1$	$x-1$	$x^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$
B	$30x^3+6x^2+x+1$	$x+1$	3

<u>Answer:</u>

Exercise 2 Are the following matrices in row echelon form ? *(the empty entries mean <u>zero</u>)*

$$A_1 = \begin{pmatrix} 4 & 1 & -1 & 1 & 0 \\ & & 4 & 1 & 1 \\ & & 1 & 0 & 0 \\ & & & & \end{pmatrix} \qquad A_2 = \begin{pmatrix} 1 & 1 & -1 & 3 & 1 & 0 \\ & & 2 & 3 & -2 & -3 \\ & & & -1 & -2 & 3 \\ & & & & & 2 \\ & & & & & \end{pmatrix}$$

Exercise 3 Consider the 2 polynomials A and B,

$$A = 2x^2 + x - 1 \qquad B = 30x^3 + 6x^2 + x + 1.$$

We have $\mathsf{Res}(A, B) = -1296$ *(sorry, I wrote 812, it was a mistake...)*

Given $n \in \mathbb{N}^\star$, we consider the map $\phi_n : \mathbb{Z}[X] \to \mathbb{Z}/n\mathbb{Z}[X]$, $\sum_i a_i X^i \mapsto \sum_i (a_i \bmod n) X^i$.

Question 1 Use Proposition 1 to compute:

$$r_5 = \mathsf{Res}(\phi_5(A), \phi_5(B)) = \mathsf{Res}(A \bmod 5, B \bmod 5)$$

Answer:

Same question for $r_3 = \mathsf{Res}(\phi_3(A), \phi_3(B)) = \mathsf{Res}(A \bmod 3, B \bmod 3)$
Answer:

Question 2 Can we use Proposition 1 (*sorry, I wrote Proposition 2*) to compute $r_2 = \mathsf{Res}(\phi_2(A), \phi_2(B)) = \mathsf{Res}(A \bmod 2, B \bmod 2)$? Compute anyway r_2.
Answer:

Exercise 4 Let R be an integral domain (Lect. II, Def. 7... like $R = \mathbb{Z}$ or $R = k[Y]$) Let $A, B, C \in R[X]$. Show the formula:

$$\mathsf{Res}(AB, C) = \mathsf{Res}(A, C)\mathsf{Res}(B, C).$$

(advice: use the formulas of Theorem 1, Slide 9)
Answer:

Exercise 5 Cf. File "`Test5-Ex6-correction.nb`" on the webpage !!!